# CIVIL-312: Hydraulic Engineering and Infrastructures

Instructor: Prof. Dr. Mirko Musa
Sustainable River Engineering, Energy, & Morphodynamics (STREEM)
GC G1 517 (Bâtiment GC)
E-mail: mirko.musa@epfl.ch

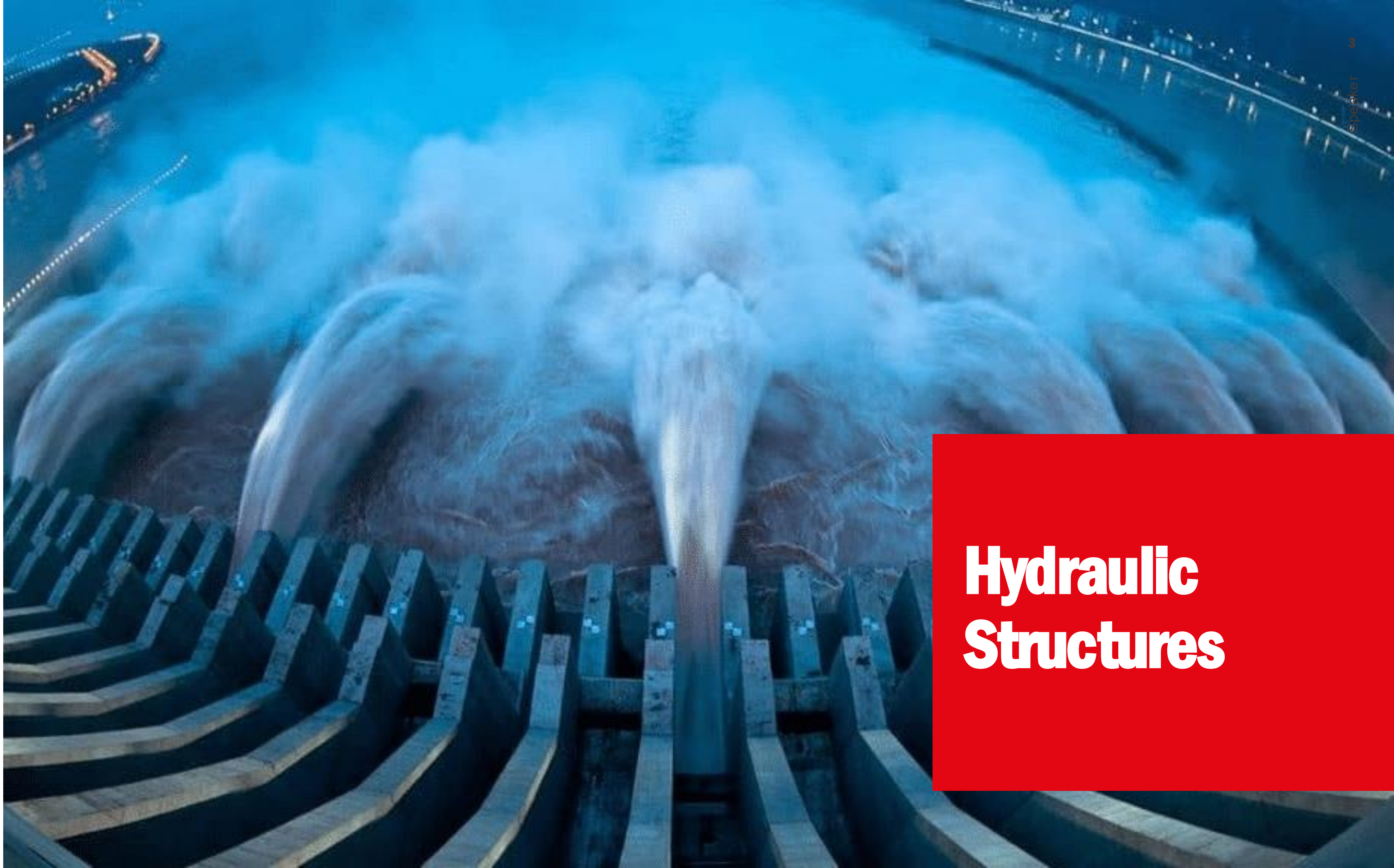
Fall 2025

# Announcements

- Late HW submissions will incur a 0.5 deduction from the final grade for every day of delay (e.g., after 2 days your maximum grade would be 5/6)

- The exercise session of Week 11 (Friday 28/11, 10:00–12:00) will be dedicated to Homework 2, which is due on Week 12, Friday 5/12 at 23:59.There will be no new exercises that day; instead, you will have the opportunity to continue working on HW2 (hopefully you will have started it beforehand) and ask us any questions.

- To give you more time, we have decided to release HW3 earlier and extend its deadline.
  - HW3 will be assigned on Week 12, Friday 5/12, and it will be due on 9 January — providing three weeks of work, excluding the holiday break.
  - Exercise session of Week 14 (Friday 14/12) will be dedicated to HW 3 development and questions



# Hydraulic Structures

# Purposes and generalities

- So far, we’ve learned how water behaves — we now understand the physical principles governing fluid motion. We have learned to describe its energy, to compute normal and critical depths, and to analyze gradually varied profiles and hydraulic jumps.
- What do we use those principles for?
- These principles form the basis of every hydraulic structure.
- Why do we design and use hydraulic structures?

**To effectively regulate, convey, and manage water—protecting communities, enabling reliable water supply and energy production, and ensuring sustainable interaction with the environment.**

# Purposes and generalities

We can broadly classify hydraulic structures in 4 groups:

1) Regulate and measure flow (e.g., weirs and gates)

2) Store or divert water (e.g., dams and spillways)

3) Dissipate energy and protect channels (e.g., energy dissipators, stilling basins, etc.)

4) Convey a specified discharge (e.g., channels and culverts)

# Purposes and generalities



We will review some examples of hydraulic structures that cover the 4 general groups and purposes.
For example:

| Structure Type | Purpose | Key Concept |
|---|---|---|
| **Weir** | Regulate discharge | Critical flow, energy head |
| **Gate** | Control water level | Pressure & momentum |
| **Spillway** | Release excess flow | Energy conversion |
| **Stilling basin** | Dissipate energy | Hydraulic jump |
| **Channel & Culvert** | Convey flow under embankment | Flow regime transitions |

# Purposes and generalities

The design and analysis of hydraulic structures require establishing relationships between flow characteristics such as discharge, pressure, flow depth and boundary shear stress, and the bounding geometry of the structure.

- Most commonly, it is required to predict the flow depth (y) around and through a structure for a particular design discharge (Q).
- Each structure has a particular relationship between the water level at and/or through the structure and the design discharge
- Both gradually varied and rapidly varied flows are possible through these structures
- 1-D methods of analysis usually are sufficient and well-developed in this branch of hydraulics.
- In several cases, a lot of empirical relationships and coefficients were established experimentally to develop design specifications

**Physics + several empirical design specifications**

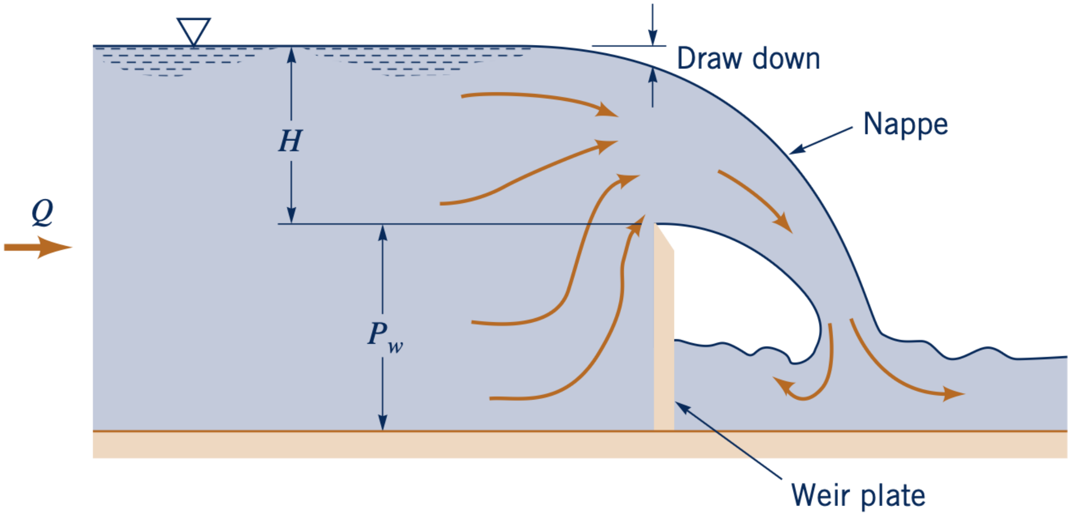
# Weirs

Speaker

# Weirs

What is a weir? https://www.youtube.com/watch?v=YkR79oDAgOg



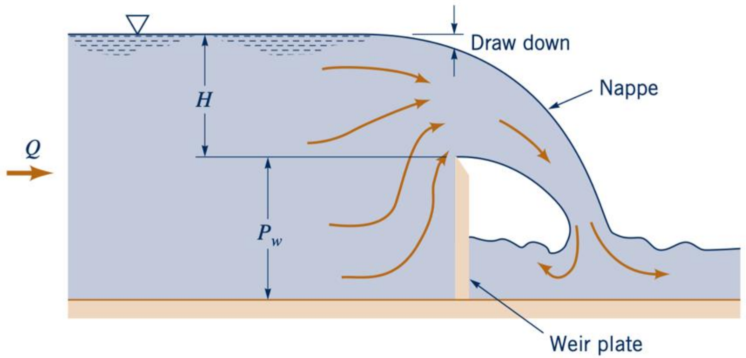



A weir is a channel obstruction over which the water must flow. It is like a small dam used to control the water level upstream.

Unlike larger dams, weirs do not create water storage but are only used **control the water level** (e.g., in stormwater-management systems and waster water plants) and **measure the discharge**.

In modern engineering, “weir” refers to **any structure allowing flow over its crest** (hydraulic control structure)

# Sharp-Crested Weirs




A ***sharp-crested weir*** is essentially a vertical sharp-edged flat plate placed across the channel in ay such that the fluid must flow across the sharp edge and drop into the pool downstream of the weir plate.

The sharp edge clearly defines the the line of separation of the ***nappe*** from the plate.

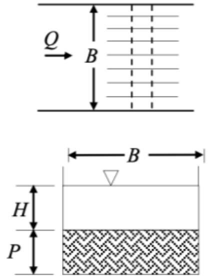
Four common types of weirs:

a) rectangular or *suppressed* weir – a weir without end walls

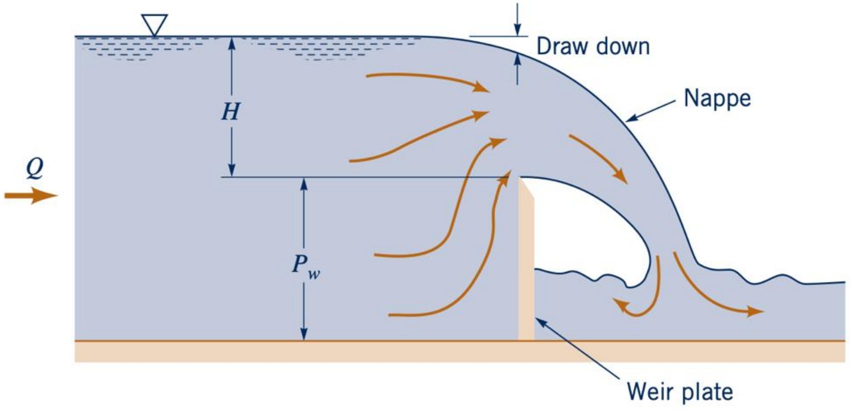
b) contracted rectangular weir (*unsuppressed*) – a weir with flow contraction due to end walls

c) Trapezoidal weir (aka, Cipolletti) with inclined to counterbalance the effects of contraction

d) Triangular (or V-notch) weir – commonly used for low flow rates. Angle $\theta$ of 60° or 90°




a. Suppressed Rectangular Weir




The complex nature of the flow over a weir makes it **impossible to obtain precise analytical expressions for the flow as a function of other parameters**, such as the weir height, $P_w$, *weir head,* $H$, the fluid depth upstream, and the geometry of the weir plate (angle $\theta$ for triangular weirs or aspect ratio, $b/H$, for rectangular weirs).

However, a relation between H and Q is derived with many simplifying assumptions.

- The main mechanism governing the flow is *gravity* and *inertia*
- In a very simplified perspective: gravity accelerates the fluid from the free-surface elevation upstream to larger velocity as it flows over the plate creating the ***nappe***
- Because of all the simplifying assumption, the final expression is adjusted using *discharge coefficients* which are inevitably determined experimentally

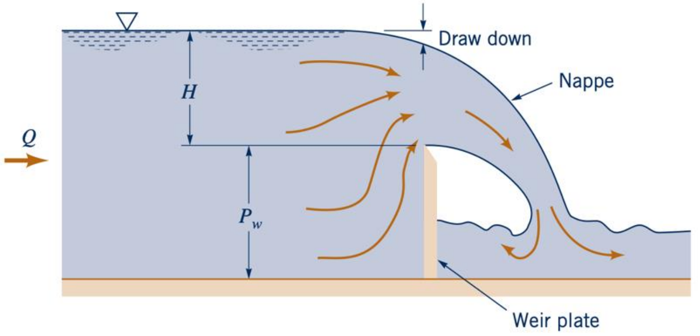
Approximations:

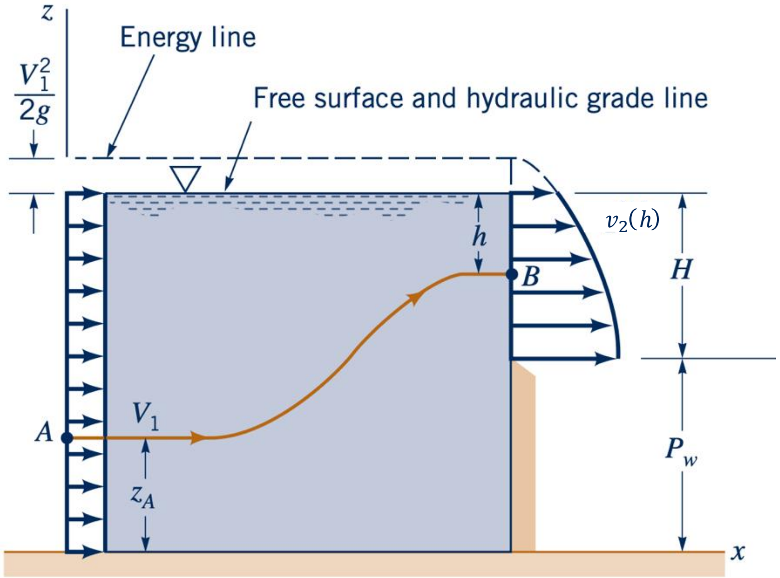
1) The velocity profile upstream of the of the weir is uniform and the pressure distribution is hydrostatic

2) All streamlines at the top of the weir are horizontal (*in reality: there is a substantial contraction of the flow jet over the plate*). So the fluid flows horizontally over the weir plate with a nonuniform velocity profile like in figure

3) The pressure within the nappe is *atmospheric*. (*in reality: the pressure is atmospheric at the top and bottom of the jet but greater than atmospheric within the jet*)

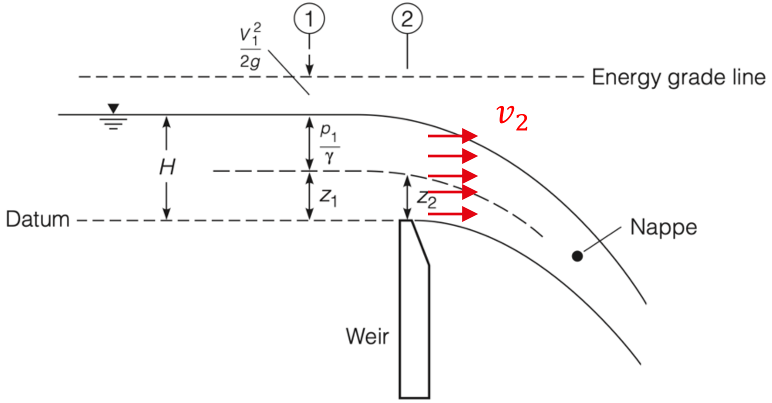
4) Viscous and surface tension effects are of secondary importance, so headlosses are neglected



Section 1, just upstream of the weir, the flow is approximately horizontal, the pressure distribution is approximately hydrostatic, so the energy is:

$$E_1 = H + \frac{V_1^2}{2g}$$




H is the elevation of the water surface above the crest of the weir and $V_1$ is the average velocity at Section 1.

In Section 2, **right on top of the crest**, we approximate the flow has horizontal and:

$$E_2 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2$$

Do this at the blackboard!

Given our assumptions before: The distribution of fluid pressure at Section 2 is such that it is equal to atmospheric pressure both at the top and bottom water surfaces, and increases above atmospheric pressure between the two water surfaces. The bottom water surface is where the water springs clear of the crest at Section 2. If the flow depth at Section 2 is small, then the pressure may be assumed equal to atmospheric pressure throughout the depth, and $E_2$ becomes

$$E_2 = \frac{v_2^2}{2g} + z_2$$

Given all these approximations and assuming no energy loss:

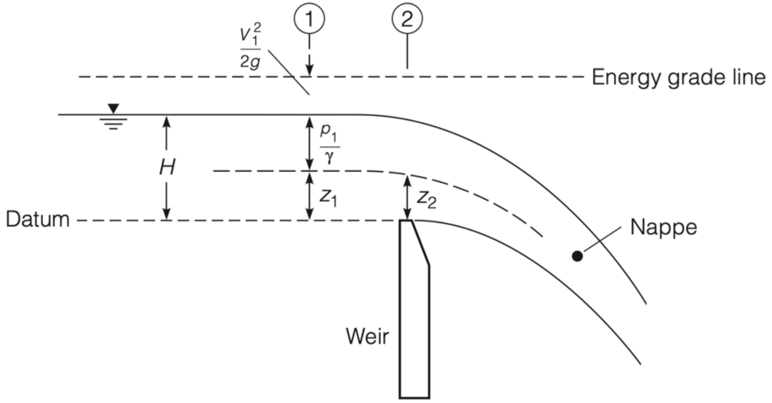
$$E_1 = E_2$$

$$H + \frac{V_1^2}{2g} = z_2 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2g\left(H - z_2 + \frac{V_1^2}{2g}\right)}$$

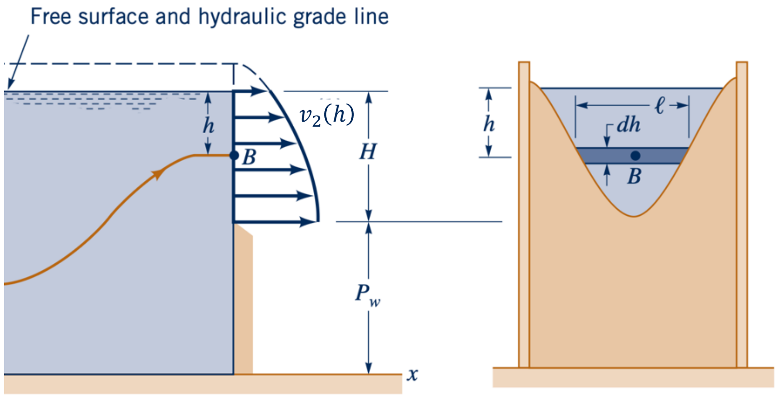
$$= \sqrt{2g\left(h + \frac{V_1^2}{2g}\right)}$$

with $h = H - z_2$




Note: if you assume that the incoming flow is negligible, $V_1 \approx 0$, the nappe velocity becomes the Torricelli velocity (flow out of an orifice):

$$v_2(h) = \sqrt{2gh}$$




So the flowrate Q can be calculated as:

$$Q = \int_{(2)} v_2 dA = \int_{h=0}^{h=H} v_2 \, \ell \, dh = 0$$

where $\ell = \ell(h)$ is the cross-channel width of the strip of the weir area (of whatever shape) as indicated in the figure to the left.

For a **rectangular weir**, $\ell$ is constant; for other weirs (e.g., triangular, trapez., circular, etc.) $\ell$ is a function of $h$

For a rectangular weir, $\ell = b$ (constant), so the discharge becomes:

$$Q = b\sqrt{2g}\int_0^H \left(h + \frac{V_1^2}{2g}\right)^{1/2} dh \;\Rightarrow Q = \frac{2}{3} b\sqrt{2g}\left[\left(H + \frac{V_1^2}{2g}\right)^{\frac{3}{2}} - \left(\frac{V_1^2}{2g}\right)^{\frac{3}{2}}\right]$$

In practice $P_w \gg H$ and so $V_1 \approx 0$, which means that $\frac{V_1^2}{2g} \ll H$ and the discharge equation simplifies to the commonly known basic rectangular weir equation

$$Q = \frac{2}{3}\sqrt{2g}\, b\, H^{3/2}$$

As we said, because of all the approximations made, a discharge coefficient $C_d$ must be determined experimentally and introduced, thus arriving to the final form:

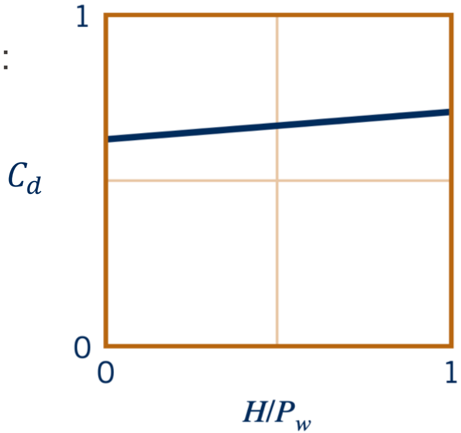
$$Q = \frac{2}{3} C_d \sqrt{2g}\, b\, H^{3/2}$$

with $C_d = f\left(Re, We, \frac{H}{P_w}\right)$ — $Re$ is the Reynolds number (viscous effects), $We$ is the Weber number (surface tension effects), and $H/P_w$ represents the effect of geometry.

Experiments have shown that $H/P_w$ is the most important variable affecting $C_d$:

$$C_d = 0.611 + 0.075\left(\frac{H}{P_w}\right)$$

which is valid for $H/P_w < 5$ and approximate up to $H/P_w = 10$.



For $H/P_w > 15$, the weir is called a *sill*, the discharge can be computed from the critical-flow equation by assuming $y_c = H + P_w$, and the discharge coefficient is given by:

$$C_d = 1.06\left(1 + \frac{P_w}{H}\right)$$

Generally, you may will find the short version of the discharge formula:

$$Q = C_w b H^{3/2}$$

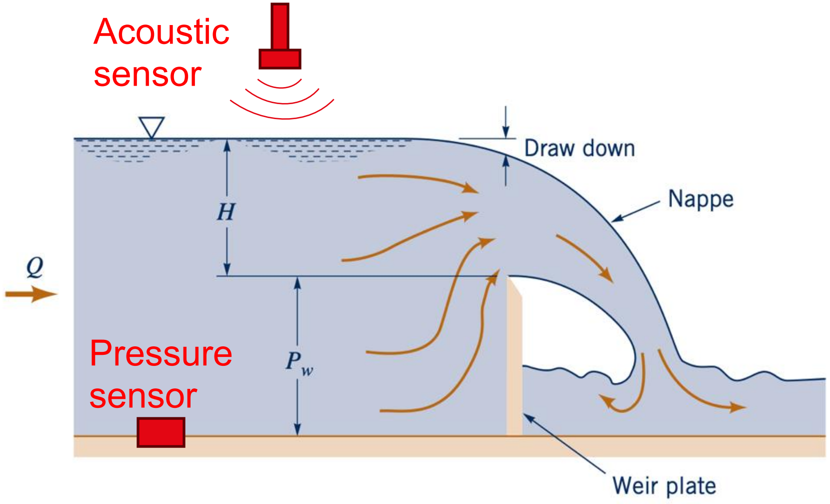
where $C_w$ is called the weir coefficient and is related to the discharge coefficient by

$$C_w = \frac{2}{3} C_d \sqrt{2g}$$

Many textbooks propose a fixed $\boldsymbol{C_d \approx 0.62}$

$$Q = C_w b H^{3/2}$$

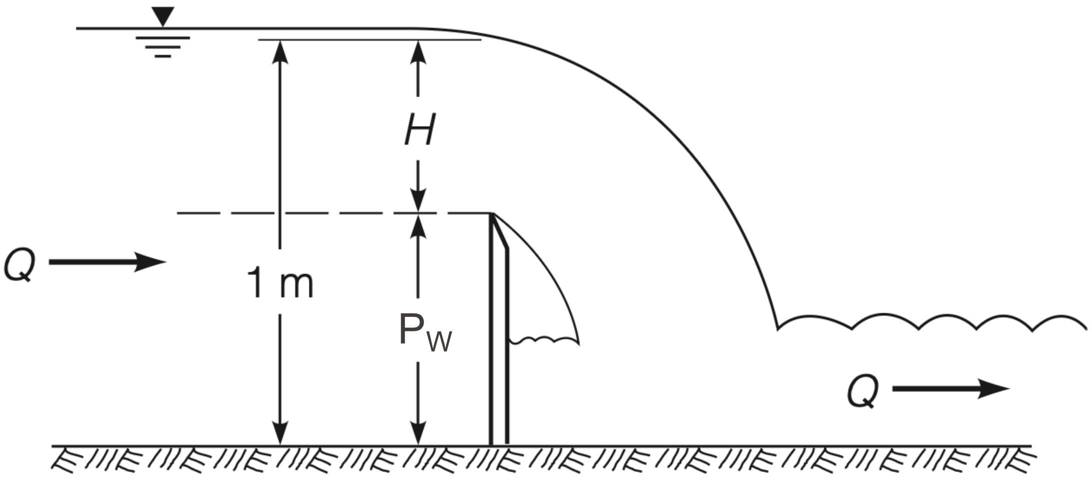
Weirs are used as a flow rate measuring device because Q only depends on the water height H before the weirs which we can measure (either through acoustic sensors to measure the water surface elevation or pressure transducers at the bottom).




Note: air is trapped underneath the nappe which tends to be entrained into the jet, thereby reducing the air pressure beneath the nappe and drawing the nappe toward the face of the weir. To avoid this effect, a vent is sometimes placed beneath the weir to maintain atmospheric pressure.

We will see that the at the lower part of the nappes is important in *spillways,* which are very similar structures to weirs

# Example

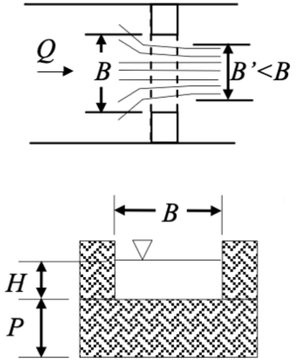



A weir is to be installed to measure flow rates in the range of 0.5–1.0 m3/s. If the maximum (total) depth of water that can be accommodated at the weir is 1 m and the width of the channel is 4 m, determine the crest height of a suppressed weir that should be used to measure the flow rate.

For a **contracted rectangular weir**: the effective length of the weir, $B'$, is less than the actual length of the weir, $B$ because the nappe is not only contracted in the vertical but also horizontally (see figure). Experiments show that the effective reduction in length is $0.10H$ to $0.20H$ when $B/H > 3$. Assuming the effective reduction in length is $0.10H$, the discharge through a contracted weir is:

$$Q = \frac{2}{3} C_d \sqrt{2g}(B - 0.1nH)H^{3/2}$$

where $C_d$ is defined using the formula introduced before and $n$ is the number of sides of the weir that are contracted (typically $n = 2$)

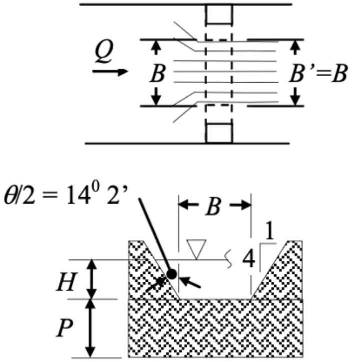



b. Contracted Rectangular Weir

For a **Cippoletti weir**: the design is made purposedly to take care of the contraction so no need for correction, B is the base of the trapezoid, and the formula is the same as before but with B:

$$Q = \frac{2}{3} C_d \sqrt{2g}\, B H^{3/2}$$

where $C_d$ is defined using the formula introduced before. If B→0, it becomes a triangular weir




c. Cippoletti Weir

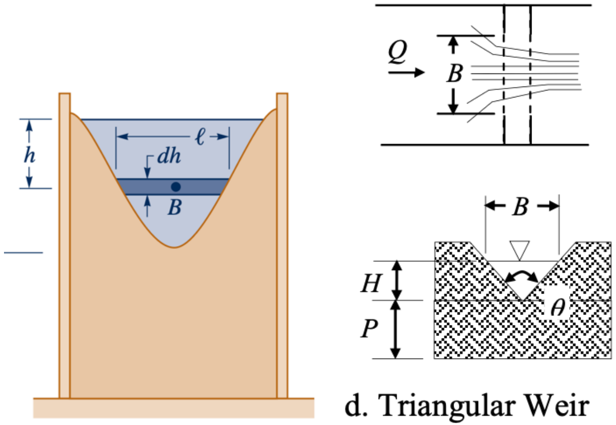
For a **triangular (V-notch) weir**, $\ell$ is a function of $h$ and is expressed as

$$\ell = 2(H-h)\tan\left(\frac{\theta}{2}\right)$$

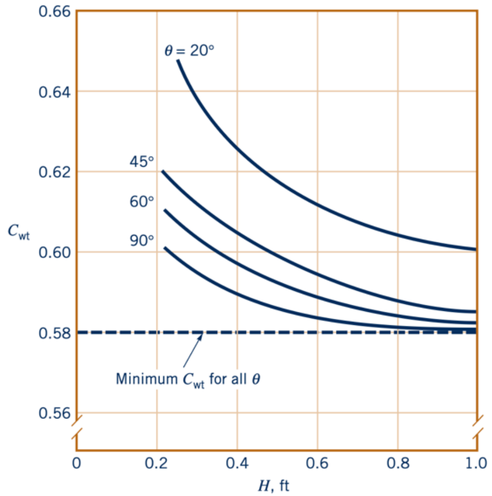
where $\theta$ is the angle of the V-notch. If we do the integration considering this expression and again neglecting the upstream velocity, we obtain:

$$Q = C_{wt}\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\,H^{5/2}$$

Typical values of $C_{wt}$ for triangular weirs are in the range of 0.58 and 0.62. Although $C_{wt}$ and $\theta$ are dimensionless, $C_{wt}$ is typically given as function of weir head $H$ which is dimensional. For example, the Army Corps of Engineers provide the relation in the figure here →

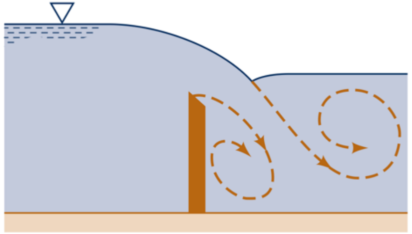


d. Triangular Weir

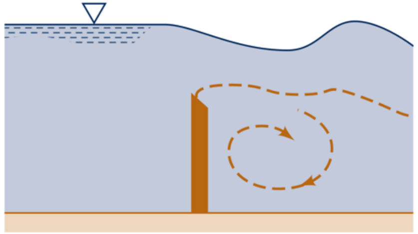


# Submerged Outflow

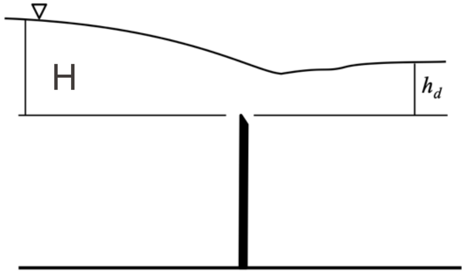
The above results for sharp-crested weirs are valid provided the area under the nappe is ventilated to atmospheric pressure.
However, depending on downstream conditions, it is possible to to have a weir that is completely submerged as shown in figure below. In this case, the expressions we derived are no longer valid.



Plunging nappe



Submerged nappe

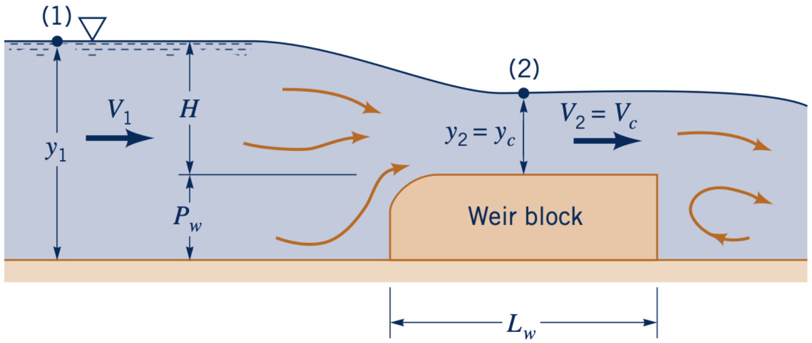



If the weir is submerged, both water levels must be measured and used in the Villemonte formula, which corrects the discharge calculate assuming free flowing:

$$Q^* = Q\left(1 - \left(\frac{h_d}{H}\right)^{3/2}\right)^{0.385}$$

where $Q^*$ is the actual discharge and $Q$ is the free discharge associated with $H$ (whereas $h_d$ is the water level downstream)

# Broad-Crested Weirs




Broad crested weirs have longitudinal lengths (i.e., along the channel) that are significantly longer that sharp crested weirs.

The length and height of the block above the channel bed are such that they induce critical flow with parallel streamlines → essentially the step problem we saw in class.

Generally, to ensure proper operation, these weirs are restricted to the range $0.08 < H/L_w < 0.5$

So the specific energy in (1) and (2) is the same, and (2) is in critical conditions:

$$H + P_w + \frac{V_1^2}{2g} = y_c + P_w + \frac{V_c^2}{2g}$$

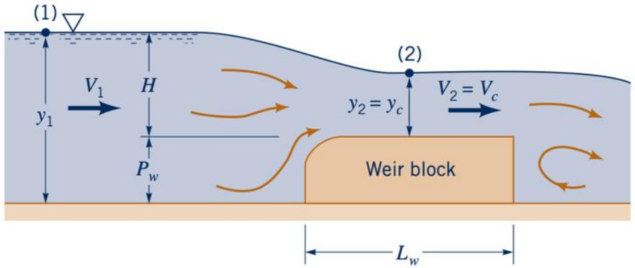
$$H - y_c = \frac{V_c^2 - V_1^2}{2g}$$

Do this at the blackboard!

We consider the incoming flow $V_1$ negligible as we did before

# Broad-Crested Weirs






Also, $Fr_c^2 = 1 \Longrightarrow \frac{V_c^2}{gy_c} = 1 \Longrightarrow V_c^2 = gy_c$

$$H - y_c = \frac{y_c}{2}$$

$$y_c = \frac{2}{3} H$$

Thus the flow rate is:

$$Q = by_2V_2 = by_cV_c = by_c(gy_c)^{1/2} = b\sqrt{g}y_c^{3/2} \Longrightarrow Q = b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}$$

Again, we need an empirical coefficient to account for the various real-world effects not included in the simplified analysis:

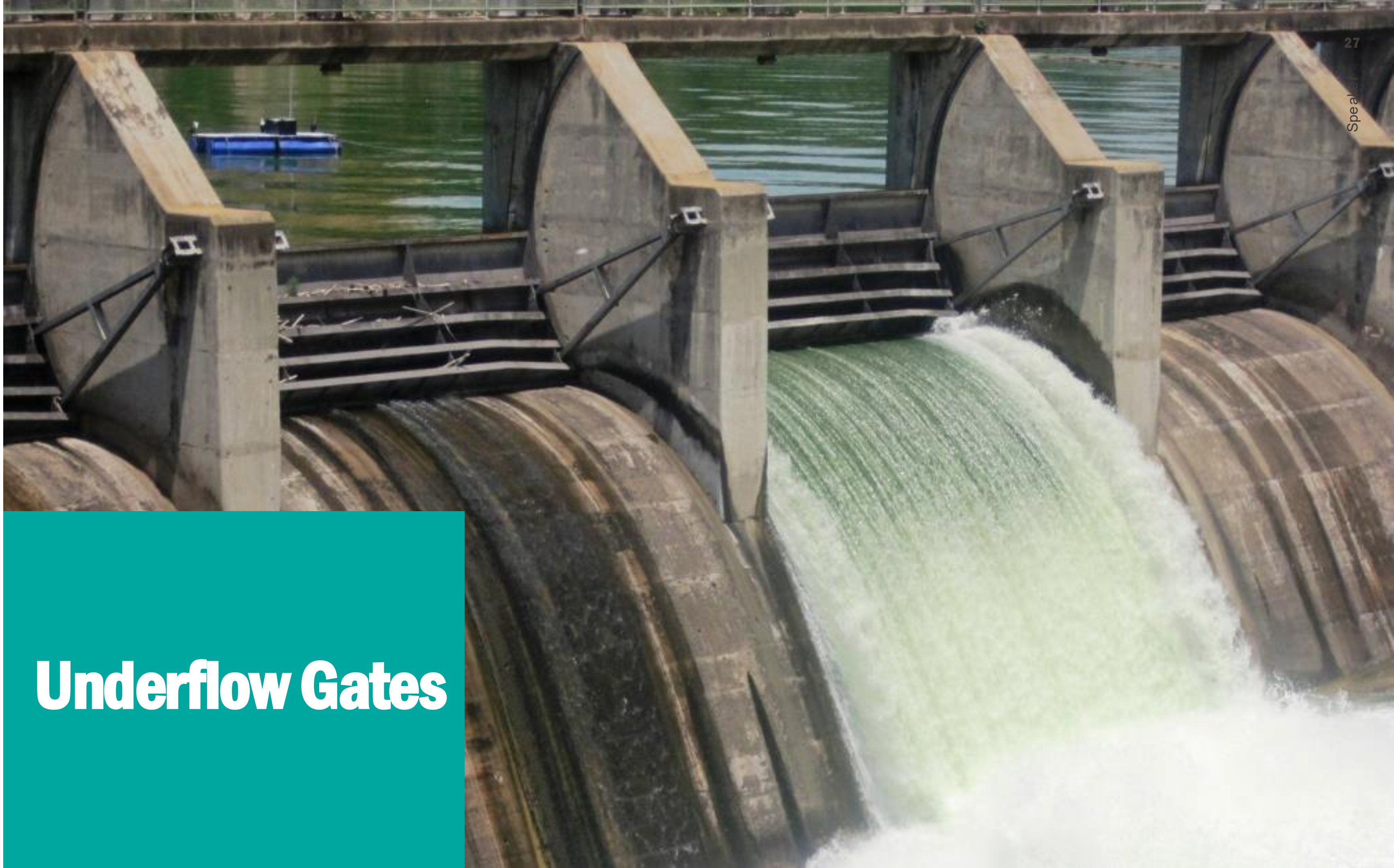
$$Q = C_{d_{bc}} b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2} \quad \text{with} \quad C_{d_{bc}} = 1.125\left(\frac{1+H/P_w}{2+H/P_w}\right)^{1/2}$$

# Example 2 – comparison between weirs



Water flows in a rectangular channel of width b = 2 m with flowrates between and Qmin = 0.02 m3/s and Qmax = 0.60 m3/s. This flowrate is to be measured by using either (a) a rectangular sharp-crested weir, (b) a triangular sharp-crested weir with θ = 90, or (c) a broad-crested weir. In all cases the bottom of the flow area over the weir is a distance Pw=1m above the channel bottom.

EPFL



# Underflow Gates

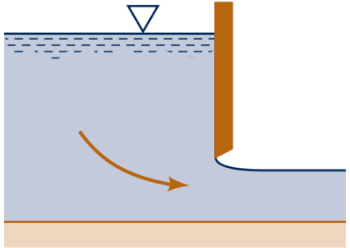
# Underflow Gates

How Different Spillway Gates Work:
https://www.youtube.com/watch?v=Ry_3icwd68o

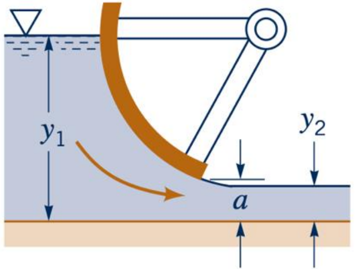
Underflow gates are structures used to control flowrate at the entrance of canals or at the crest of a spillway.

Mainly three general types:

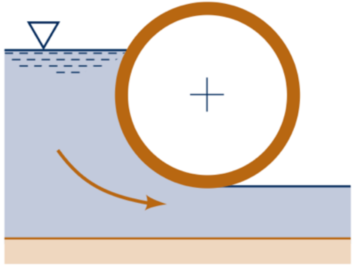
- Vertical gates (also referred to as sluice gate) – very common, especially in canals. They slide up and down using roller and track assemblies.
- Radial gates (also known as Tainter) – rotate around an axis; very common at the crest of spillways.
- Drum gates – long , hollow, cylindrical (drum-shaped) structure, typically made of steel



Vertical Gate




Radial (Tainter) Gate

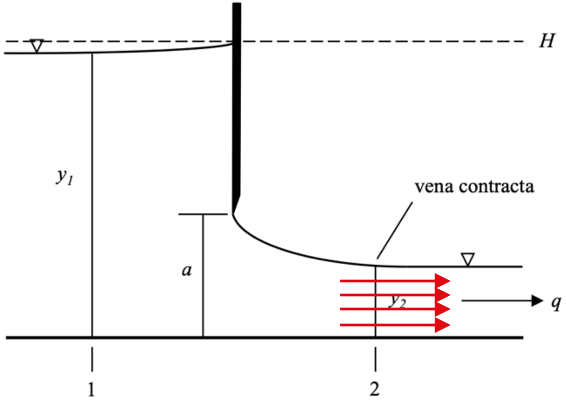


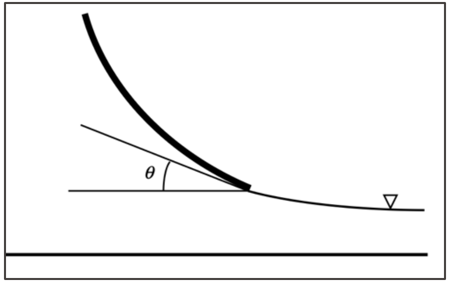
Drum Gate

The flow under a gate is said to be **free outflow** when the fluid issues as a jet of supercritical flow with a free surface open to the atmosphere (unsubmerged); **in this case, gates are true control**. The outflow may also be submerged and the gate will not be the control point.

# Unsubmerged underflow gates

The unsubmerged analysis is straightforward and based on energy conservation. The gate is a control; the flow upstream is subcritical, and the flow downstream is supercritical




Like for the weir, the flow exit the aperture with a curvature called "vena contracta". In this case though is limited only on one side, since the other is constrained by the channel bottom.

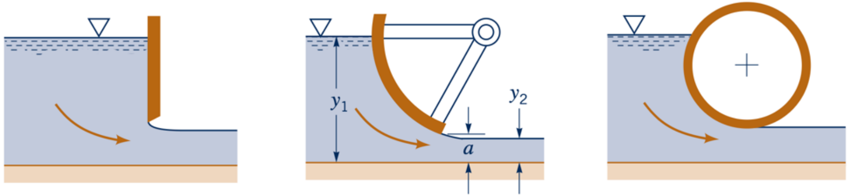
The flow depth at the vena contracta is defined by the geometry of the opening, and can be estimated as:

$$y_2 = C_c a$$

For $\frac{y_1}{a} < 0.7$, $C_c \approx 0.61$ which is the value commonly used. $C_c$ depends on the inclination of the lip of the gate as

$$C_c = 1 - 0.75\theta + 0.36\theta^2$$

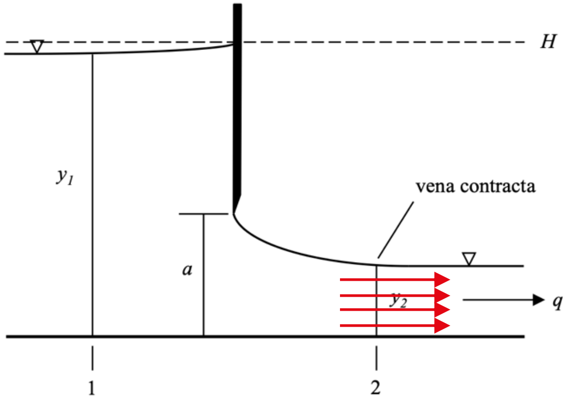
so it increases as $\theta$ decreases. For radial gates is more than 0.61 and for drum is close to 1.



# Unsubmerged underflow gates



Energy losses can be neglected because the flow lines are considered parallel and friction is negligible over short distances. So we can apply energy conservation between (1) and (2):




$$E_1 = E_2$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

with:

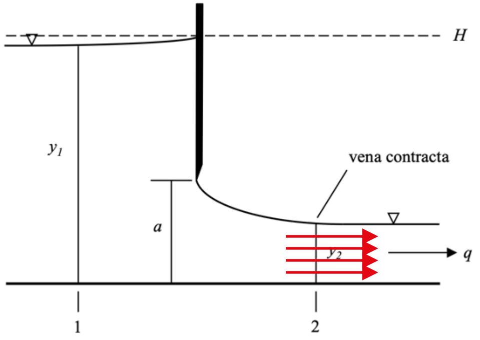
- vena contracta → $y_2 = C_c a$
- Continuity → $Q = b y_1 V_1 = b C_c a V_2$

$$y_1 + \frac{V_2^2}{2g}\left(\frac{aC_c}{y_1}\right)^2 = aC_c + \frac{V_2^2}{2g}$$

Do this at the blackboard!

# Unsubmerged underflow gates






$$y_1 + \frac{V_2^2}{2g}\left(\frac{aC_c}{y_1}\right)^2 = aC_c + \frac{V_2^2}{2g} \rightarrow \frac{V_2^2}{2g}\left(\frac{aC_c}{y_1}\right)^2 - \frac{V_2^2}{2g} = aC_c - y_1$$

$$\frac{V_2^2}{2g}\left[\left(\frac{aC_c}{y_1}\right)^2 - 1\right] = y_1\left(\frac{aC_c}{y_1} - 1\right) \rightarrow V_2 = \sqrt{2gy_1}\frac{1}{\sqrt{\left(\frac{aC_c}{y_1}\right) + 1}}$$

So: $$Q = by_2V_2 = b\,a\,C_c\sqrt{2gy_1}\frac{1}{\sqrt{\left(\frac{aC_c}{y_1}\right) + 1}} \rightarrow q = \frac{Q}{b} = C_d a\sqrt{2gy_1}$$ with $C_d = \frac{C_c}{\sqrt{\left(\frac{aC_c}{y_1}\right)+1}}$

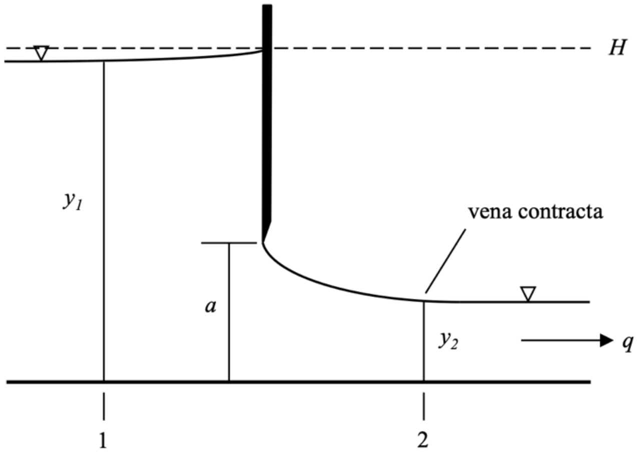
$C_d$ is therefore a function of the contraction coefficient $c_c = y_2/a$ and the depth ratio $y_1/a$.

For unsubmerged free outflow typical values of the discharge coefficient are on the order of 0.55 to 0.60.

# Examples



## Example 1

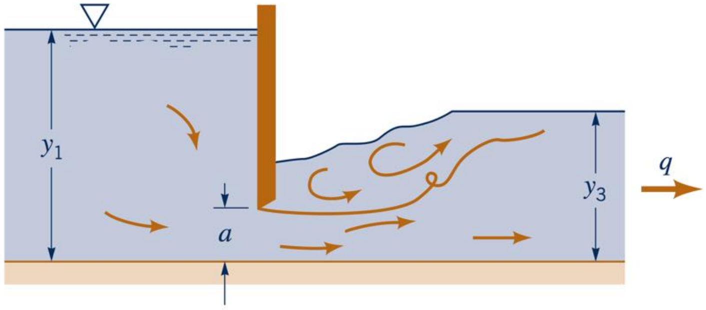



A 4.3 m wide canal has an upstream depth of 2.7 m. A 1.5 m wide sluice gate is opened 45 cm. After contracting, the downstream depth is 27.5 cm.

What is the discharge exiting through the gate?

# Submerged underflow gates






If the water level downstream is controlled by some other obstacle downstream, the flow might be subcritical. If the momentum of the flow downstream dominates the momentum of the outflow from the gate, we will have a downed HJ and the outflow will be submerged: the jet of water issuing from under the gate is overlaid by a mass of turbulent water.

In other words, to check whether we have submerged outflow we look at the conjugate depth of the vena contracta we would have (if it wasn't submerged):

*If the tailwater depth (y3) is less than the conjugate depth of the vena contracta, the flow through the gate will be free, and if the tailwater depth is greater than the conjugate depth of the vena contracta, the flow through the gate will be submerged.*

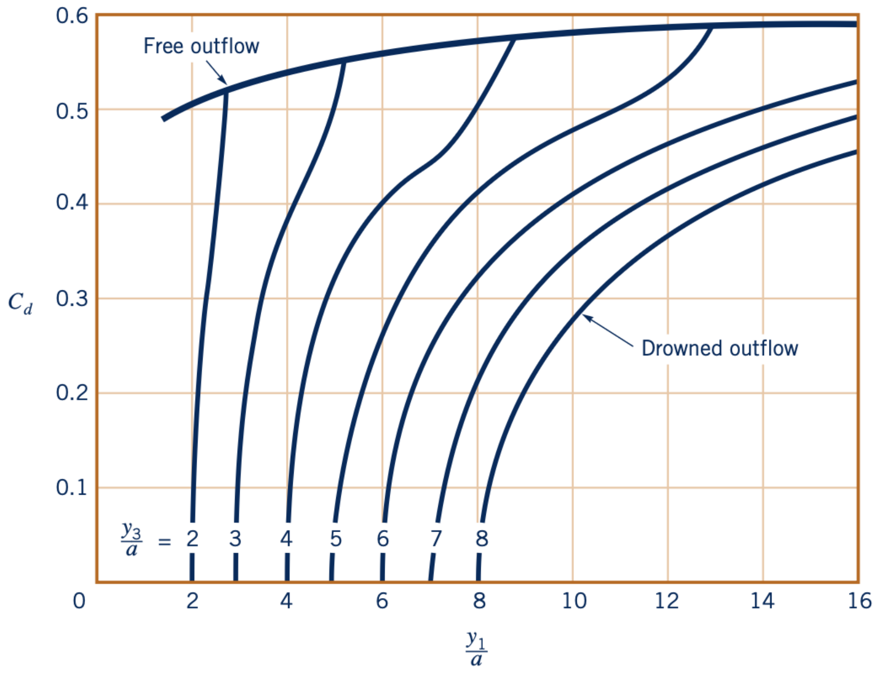
Let's see this in Example 2

# Submerged underflow gates



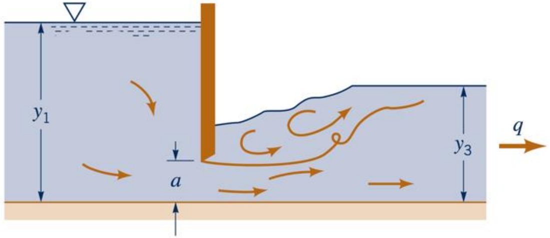
The flowrate for drowned gate can be obtained from the same equation that is used for free outflow but the discharge coefficient is modified appropriately. The values of $C_d$ for drowned outflow cases are indicated as the series of lower curves in the plot here below.

The maximum value of $C_d$ is indicated by the top line and corresponds to free outflows. For values of $y_3/a$ that give $C_d$ values between zero and its maximum, the jet from the gate is drowned by the downstream water and the flowrate is therefore reduced when compared with a free discharge situation.



# Examples

Example 2




Let’s calculate the gate aperture such that we PREVENT the downing of the hydraulic jump downstream of the gate, so that the gate is not submerged.

1. Calculate the normal depth downstream based on the stream characteristics and Manning equation
2. Find the critical depth. Determine if the slope is mild (i.e., if it can support a hydraulic jump)
3. Maximum gate opening to prevent drowning.